FILM COOLING BY INJECTION INTO A TURBULENT BOUNDARY LAYER

E. P. Volchkov, E. G. Zaulichnyi, S. S. Kutateladze, and A. I. Leont'ev

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 2, pp. 103-106, 1967

ABSTRACT: Most papers on film cooling concern injection of a homogeneous gas. Stollery et al. [1] examined the case of tangential injection of gas into a boundary layer, the specific heat c_{p_s} differing little from that of the main flow, c_{p_0}.

Here we examine the effectiveness of film cooling of a thermally isolated planar wall by local supply to a turbulent boundary layer.

§1. If we neglect thermal and pressure diffusion and also heat conduction due to diffusion, the energy equation for the boundary layer on a planar wall can [2] be put as

$$\rho w_x \frac{\partial i}{\partial x} + \rho w_y \frac{\partial i}{\partial y} = -\frac{\partial q}{\partial y} . \qquad (1.1)$$

If the Prandtl number P = 1 and the Lewis number L = 1, we have for the density of the heat flux that

$$q = -\frac{\Lambda}{C_p}\frac{\partial i}{\partial y} \qquad \left(i = \int_0^T c_p dT + i^\circ\right) \qquad (1.2)$$

in which i° is the heat of formation of a component and C_p is the heat capacity of the gas mixture.

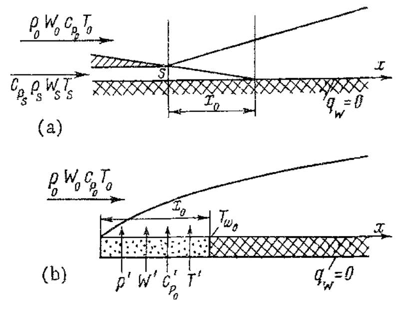

Fig. 1

Integration of (1.1) over the thickness of the enthalpic boundary layer, with the concept of the total-energy thickness

$$\delta_i^{**} = \int_0^{\delta_i} \frac{\rho w}{\rho_0 w_0}\left(\frac{i - i_0}{i_w - i_0}\right) dy \qquad (1.3)$$

gives us an integral relation for the energy of the boundary layer:

$$\frac{dR_i^{**}}{dX} + \frac{R_i^{**}}{\Delta i}\frac{d\Delta i}{dX} - \frac{j_w}{\rho_0 w_0} R_L = R_L \frac{q_w}{\rho_0 w_0 \Delta i} ,$$

$$R_i^{**} = \frac{\rho_0 w_0 \delta_i^{**}}{\mu_0} , \qquad X = \frac{x}{L} ,$$

$$R_L = \frac{\rho_0 w_0 L}{\mu_0} , \qquad \Delta i = i_w - i_0 , \qquad (1.4)$$

in which j_w is the transverse flux of material at the wall.

Consider the turbulent boundary layer on an impermeable wall beyond the region of supply of the second material (Fig. 1), when the heat flux through the wall is $q_w = 0$. The cooled gas is supplied via a tangential slot (Fig. 1a) or an initial porous part (Fig. 1b). If the cooling agent is a liquid that evaporates, or if reactions occur on the first part of the plate, the action of the gaseous products is similar to that of supply of a second gas.

We determine the temperature of the isolated wall by a method previously described [2, 3].

We integrate (1.4) from x_0 to x with j = 0 and $q_w = 0$ to get

$$R_i^{**}\Delta i = R_{i_0}^{**}\Delta i_0, \quad \theta_i = \frac{i_0 - i_w^*}{i_0 - i_{w_0}} = \frac{R_{i_0}^{**}}{R_i^{**}} = \frac{\delta_{i_0}^{**}}{\delta_i^{**}} ,$$

$$(\theta_i = 1 \;\text{ for }\; 0 < x < x_0), \qquad (1.5)$$

in which i_w^* is the total enthalpy of the gas mixture at the wall, while i_{w0} and $\delta_{i_0}^{**}$ are the total enthalpy of the gas mixture at the wall and the total-energy thickness at $x = x_0$.

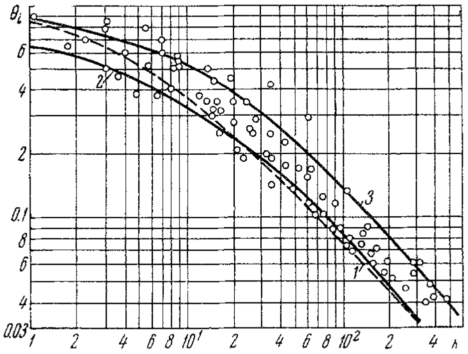

Fig. 2

The rate of turbulent mixing is maximum near the wall (but not in the viscous sublayer), where $\partial w_x/\partial y \to$ max; equalization of the flow parameters will therefore occur more rapidly in the wall section of the boundary layer, and we can [2, 3] for $x \to \infty$ write the following equation for the quasi-isothermal flow with a power-law approximation for the velocity profile (n = 1/7):

$$\beta = \frac{\delta_i^{**}}{\delta^{**}} \to \beta_{\max} = 9, \qquad \left(\delta^{**} = \int_0^{\delta} \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{w}{w_0}\right) dy\right), \qquad (1.6)$$

with $\rho \to \rho_w \to \rho_0$, while δ^{**} is the momentum thickness in the boundary layer, which is found from the solution of the momentum

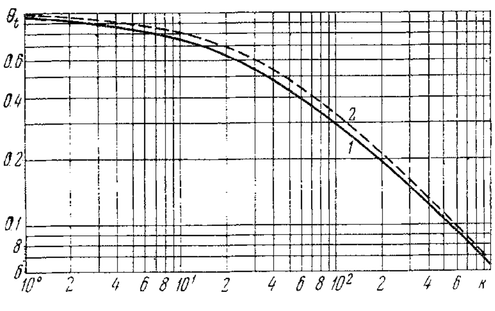

Fig. 3

equation for a planar plate [3], and for $x \to \infty$ for a quasi-isothermal flow of an incompressible fluid:

$$R^{**} = [A(m+1)R_x]^{1/(m+1)}, \qquad (R^{**} = \rho_0 w_0 \delta^{**}/\mu_0), \qquad (1.7)$$

in which A and m are the coefficient and power in the power-law approximation for the law of friction (A = 0.0128 and m = 0.25 for a power-law profile with n = 1/7).

From (1.5)-(1.7) we get the interpolation formula

$$\theta_i = [1 + 0.24 R_{\Delta x}/R_{i_0}^{**1.25}]^{-0.8} . \qquad (1.8)$$

The Reynolds number $R_{i_0}^{**}$ in the initial part is found from the energy equation for that part in the case of supply of the coolant via

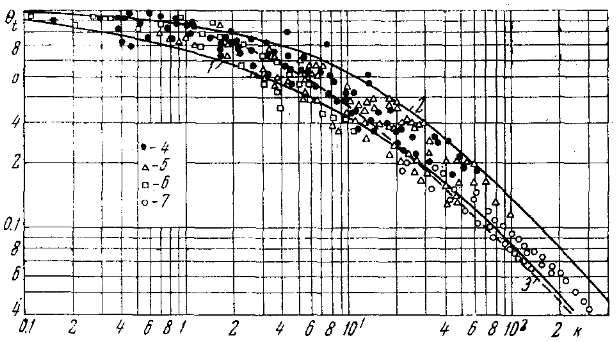

Fig. 4

an initial porous part, or when there are chemical reactions or phase transitions there.

If gas is injected via a tangential slot [4]

$$R_{i_0}^{**} = \frac{\rho_s w_s s}{\mu_0} = R_s \frac{\mu_s}{\mu_0}. \tag{1.9}$$

We see from (1.8) that the thermal efficiency (in terms of the total enthalpies) is the same when the gases are different as when they are the same. If we use the principle of superposition of temperature distributions, we get formulas similar to those previously derived [4]:

$$\theta_i = \left\{\left[1 + \frac{62.5}{k + 0.143}\right]^{0.114} - 1\right\}^{0.8} [1 + 0.016k]^{-0.16}$$

$$\text{for} \quad \frac{w_s}{w_0} \ll 1, \tag{1.10}$$

$$\theta_i = \left\{\left[1 + \frac{62.5}{k+2}\right]^{0.2} - 1\right\}^{0.8} [1 + 0.016k]^{-0.16}$$

$$\text{for} \quad \frac{w_s}{w_0} \approx 1, \qquad \left(k = \frac{R_{\Delta x}}{R_s^{1.25}} \left(\frac{\mu_0}{\mu_s}\right)^{1.25}\right). \tag{1.11}$$

Curves 1–3 of Fig. 2 are derived, respectively, from (1.8), (1.10), and (1.11) for helium injected via a tangential slot into a flow of air; the points are the experimental results of Papell and Hatch [1], and the agreement with the theory is satisfactory. In practical calculations, it is necessary to determine the wall temperature, for which we need to know the heat capacity of the gas mixture at the wall.

The equation for mass transfer, without heat and pressure diffusion, is

$$\rho w_x \frac{\partial K}{\partial x} + \rho w_y \frac{\partial K}{\partial y} = \frac{\partial}{\partial y}\left(\rho D \frac{\partial K}{\partial y}\right), \tag{1.12}$$

in which K is the total concentration of the injected component. It follows from (1.1) and (1.12) that the distribution of the total enthalpy resembles that of the weight concentration when the boundary conditions are similar. Then

$$\theta_i = \frac{i_0 - i_w^*}{i_0 - i_{w_0}} = \frac{K_0 - K_w^*}{K_0 - K_{w_0}}, \quad \text{or} \quad K_w^* = K_0 - \theta_i (K_0 - K_{w_0}) \tag{1.13}$$

in which K_w^* is the concentration of the injected component at the wall in the present section and K_{w_0} is that concentration at $x = x_0$.

The heat capacity of the mixture at the wall is

$$c_{pw}^* = c_{p_s} K_w^* + c_{p_0}(1 - K_w^*) = c_{p_0} + (c_{p_s} - c_{p_0}) K_w^*. \tag{1.14}$$

From (1.14) and the expression for θ_i,

$$\theta_i = \frac{c_{p_w}{}^* T_w{}^* - c_{p_0} T_0}{c_{p_{w_0}} T_{w_0} - c_{p_0} T_0}, \tag{1.15}$$

we get

$$\theta_t = \frac{T_w{}^* - T_0}{T_{w_0} - T_0} =$$

$$= \frac{\theta_i (c_{p_{w_0}} T_{w_0} - c_{p_0} T_0) - (c_{p_{w_0}} - c_{p_0}) T_0 K_w{}^*}{[c_{p_0} + (c_{p_{w_0}} - c_{p_0}) K_w{}^*] (T_{w_0} - T_0)}, \tag{1.16}$$

in which K_w^* is found from (1.13). For a different gas injected through a tangential slot,

$$K_0 = 0, \quad K_{w_0} = 1, \quad T_{w_0} = T_s, \quad c_{p_{w_0}} = c_{p_s},$$

and from (1.13) and (1.16) we get

$$\theta_t = \frac{\theta_i c_{p_s}}{\theta_i (c_{p_s} - c_{p_0}) + c_{p_0}}. \tag{1.17}$$

This formula agrees with that previously derived [1] for this case.

§2. Let the energy thickness be represented [3] by

$$\delta_T{}^{**} = \int_0^{\delta_T} \frac{c_p \rho w}{c_{p_0} \rho_0 w_0} \left(1 - \frac{T - T_w}{T_0 - T_w}\right) dy. \tag{2.1}$$

Then the integral relation for the energy is written in terms of the temperature [3]:

$$\frac{d\delta_T{}^{**}}{dx} + \frac{\delta_T{}^{**}}{\Delta T} \frac{d(\Delta T)}{dx} - \frac{j_w c_p'}{\rho_0 w_0 c_{p_0}} = \frac{q_w}{c_{p_0} \rho_0 w_0 \Delta T}. \tag{2.2}$$

This equation is correct only if the two gases do not differ greatly in c_p. Then, operating on (2.2) as for (1.4), we get formulas for θ_t analogous to (1.8), (1.10), and (1.11), except that $R_{i_0}^{**}$ is replaced by

$$R_T{}^{**} = R_s \frac{c_{p_s}}{c_{p_0}} \frac{\mu_s}{\mu_0}, \tag{2.3}$$

and

$$k = \frac{R_{\Delta x}}{R_s^{1.25}} \left(\frac{\mu_0 c_{p_0}}{\mu_s c_{p_s}}\right)^{1.25}. \tag{2.4}$$

Curve 1 of Fig. 3 shows the enthalpy calculated from (1.8) and (1.9), while curve 2 shows that from (1.8) with (2.3) for helium injected into air. It is clear that the results differ little even when $c_{p_s}/c_{p_0} = 5.2$.

Curves 1–3 of Fig. 4 show θ_t given by (1.10), (1.11), and (1.8) in conjunction with (2.3) and (2.4), as well as results for slot cooling by injection for $0 < w_s/w_0 < 1$: points 4 are for helium in air [1], points 5 are for air in air at $T_s/T_0 \approx 0.6$ [1], points 6 are for air in air at $T_s/T_0 \approx 0.3$ [5], and points 7 are for air in air at $T_s/T_0 \approx 1$ [6].

REFERENCES

1. I. L. Stollery and A. A. El-Ehwany, "A note on the use of a boundary-layer model for correlating film-cooling data," Internat. J. Heat and Mass Transfer, vol. 8, no. 1, 1965.

2. S. S. Kutateladze and A. I. Leont'ev, "Thermal curtain for a turbulent boundary layer of gas," Teplofizika Vysokikh Temperatur, no. 2, 1963.

3. Friction and Heat and Mass Transfer in a Turbulent Boundary Layer, edited by S. S. Kutateladze [in Russian], Izd. SO AN SSSR, 1964.

4. E. P. Volchkov and V. Ya. Levchenko, "Effectiveness of gas screen in a turbulent boundary layer," PMTF [Journal of Applied Mechanics and Technical Physics], no. 5, 1965.

5. V. Ya. Borodachev, Theoretical and Experimental Study of Air Boundary Cooling of a Planar Plate [in Russian], Oborongiz, 1956.

6. R. A. Seban, "Heat transfer and effectiveness for a turbulent boundary layer with tangential fluid injection," Trans. ASME, C, vol. 82, no. 4, 1960.

18 October 1965 Novosibirsk